COMPARING MEASURED WITH SIMULATED VERTICAL SOIL STRESS UNDER VEHICLE LOAD: ARE THE MEASUREMENTS OR THE MODELS WRONG?

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Abstract

The load transfer within agricultural soil is typically modelled on the basis of the theory of stress transmission in elastic media, usually in the semi-empirical form that includes the “concentration factor” (v). Measurements of stress in soil are needed to evaluate model calculations, but may be biased because transducers do not read true stresses.
The aim of this paper was to measure and simulate soil stress under defined loads. First, we investigated the accuracy of the transducers *in situ* by measuring stress at high spatial and temporal resolution at 0.1 m depth under a known load. Stress in the soil profile at 0.3, 0.5 and 0.7 m depth was measured during wheeling at field capacity on five soils (13-66% clay). Stress propagation was then simulated with the semi-analytical model, using vertical stress at 0.1 m depth estimated from tyre characteristics as upper boundary condition, and v was obtained at minimum deviation between measurements and simulations.
The transducer readings over-predicted the true vertical stress by 10%. Consequently, the measured stresses were corrected before further analysis. For the five soils, we obtained an average v of 3.9 (for stress propagating from 0.1 to 0.7 m depth). This was not significantly different from v = 3, i.e. v for homogenous, isotropic and linear-elastic material. We noted that v was strongly dependent on the accuracy of stress measurements, and on the upper stress boundary condition used for simulations. Finite element simulations indicate that for an elasto-plastic layered soil (topsoil over plough pan over subsoil) propagation of vertical stresses is not appreciably different from that in a homogeneous isotropic and linear-elastic soil unless layers with (unrealistically) high soil stiffness are considered.
Our results highlight the importance of accurate stress readings and realistic upper model boundary conditions, and suggest that actual stress propagation was in line with predictions according to elastic theory for the conditions investigated.

Keywords: Stress propagation; Elastic material; Concentration factor; Modelling

Introduction

The transmission of stress (stress propagation) within soil is of major importance since the soil can undergo deformation due to stress, resulting in changes in soil functions. Knowledge of stress transmission is needed for two purposes, among others: first, in order to understand the relationships between cause (soil stress due to mechanical loading) and effect (changes in soil pore functioning); and second, to develop prediction models and decision support tools that can help farmers in prevention of soil compaction.
Stress propagation in agricultural soil is typically modelled based on the problem of a normal load of the surface of a homogeneous isotropic elastic halfspace,

for which the analytical solution is due to Boussinesq (1). Most often the equation by Fröhlich (2) is used, which allows the alteration of the decay pattern of the vertical stress due to Boussinesq's solution by introducing a "concentration factor". Following this approach, stress at any depth, *z*, due to loading at the soil surface, can be calculated as follows. The contact area is divided into *i* small elements with an area A_i each and a normal stress, σ_i, carrying the load $P_i = \sigma_i A_i$, which is treated as a point load. Disregarding horizontal stresses in the contact area, vertical stress, σ_z, is then calculated as (3):

$$\sigma_z = \sum_{i=0}^{i=n} \left(\sigma_z\right)_i = \sum_{i=0}^{i=n} \frac{\nu P_i}{2\pi z_i^2} \cos^{\nu+2} \theta_i \qquad \text{(Eq. 1)}$$

where θ_i is the angle between the normal load vector and the position vector from the point load to the desired point, and *v* is the concentration factor (2). For *v* = 3, Eq. (1) satisfies the elastic theory-based solution by Boussinesq (1). Hence, for a given surface load, vertical stress a depth *z* becomes a sole function of *v* (Eq. 1).

The concentration factor was introduced because the rate of decay of the stress as predicted by the classical theory of elasticity (i.e., *v* = 3, Eq. (1)) was found to be at variance with experimental observations of vertical stress distributions in soil (4). The discrepancy between simulated and measured stress was ascribed to inaccurate model predictions, while measured stress values were assumed to be correct. However, measurements of stress in soil may be biased because transducers do not read true stresses (5, 6). Stress simulations, e.g. using Eq. (1), are sensitive to the upper model boundary condition, i.e. the area over which the stress is applied and the distribution of the surface stresses (7).

The aim of this paper was to measure and simulate soil stress under defined loads. First, the accuracy of the transducers used for measurements of soil stress was investigated. Second, measured stress was compared with simulated stress using Eq. (1), and simulations using Eq. (1) were compared with finite element calculations. Third, the sensitivity of *v* (Eq. (1)) to the upper model boundary condition was investigated.

Materials and methods

Accuracy of stress transducers

We investigated the accuracy of the transducers *in situ* by measuring stress at high spatial and temporal resolution at 0.1 m depth under a known load. We investigated several transducer types (8). In this paper, we focus on the transducer developed by Arvidsson & Andersson (9), because this was used for measurements of soil stress (see next section).

We selected the headland of an experimental field (sandy loam, water content close to field capacity) that had not been tilled for several years, and which had a density comparable to that of the subsoil (data not shown), which is important for the calibration of transducers normally installed in the subsoil. The soil was rotary tilled to 0.1 m before the test. The rotovated soil was removed, and a trench was excavated that matched the geometry of the transducer houses. Several transducers across the driving direction were carefully installed into the undisturbed, dense soil in the trenches, and soil was refilled on top of the transducers to the level of the surrounding soil surface. Measurements were made in 2 µs bursts at 2 kHz as described in Schjønning et al. (10). We used a specially constructed tractor-towed trailer to apply the load without disturbing the soil with the tractor wheels. Driving speed was 1.9 km h^{-1}. The tyre was a 800/50R34 without lugs, loaded with 29.5 kN and a rated inflation pressure of 50 kPa. Transducers were installed twice on two different plots, and ten drives were performed for each installation.

Then, we calculated the wheel load based on the measured stress, i.e. the apparent wheel load, F_{app}, as

$$F_{app} = \int_A \sigma_z dA \qquad \text{(Eq. 2)}$$

where *A* is the contact area, and σ_z is the measured vertical stress. The deviation of measured stresses from the true stresses was evaluated by comparing the wheel load recorded on a weighbridge, F_{wheel}, with F_{app}.

Measurements of vertical soil stress

We used experimental data of measured vertical soil stress from wheeling experiments done on five soils (13-66% clay; Table 1). All fields (Table 1) have been conventionally tilled, including annual mouldboard ploughing to a depth of about 0.25 m. Experiments were carried out in autumn before primary tillage or in spring (i.e. about half a year after primary tillage). Most experiments were done with several wheel loads and/or tyre inflation pressures (Table 1). Driving speed was typically 2 m s^{-1}. Wheeling experiments were carried out at about field capacity (11). During wheeling experiments, vertical stress was measured by installing probes into the soil horizontally from a dug pit (9, 12). Stress was measured at three different depths, namely at 0.3, 0.5 and 0.7 m. In this study, we used vertical stress measured below the loading centre.

Some of the wheeling experiments have been reported earlier (12, 13, 14). In the present study, we collated these data and analyzed them with respect to stress propagation.

Simulation of soil stress

We simulated vertical soil stress by means of Eq. (1) for the given situations using *SoilFlex* (15). Because the upper model boundary condition (i.e. the tyre-soil contact) was not measured during all experiments listed in Table 1, it was estimated from tyre and loading characteristics with the model of Keller (16) as incorporated in *SoilFlex*. Because the model by Keller (16) was based on measurements at 0.1 m depth (close to the tyre-soil contact), we used estimates by this model as input to Eq. (1) at 0.1 m depth. That is, stress propagation was simulated from 0.1 m and down.

A subroutine was programmed in *SoilFlex* that yields the root mean square error (RMSE) between measured and simulated stress as a function of *v*. The RMSE is given as:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\hat{\sigma}_z - \sigma_z\right)^2} \qquad \text{(Eq. 3)}$$

where n is the number of observations, $\hat{\sigma}_z$ is the predicted vertical stress, and σ_z is the measured vertical stress. For further analysis we used (the optimum) *v* at minimum RMSE. From the different loading situations on one soil (see Table 1), we calculated an average *v* for each soil. We also calculated the bias for each measuring depth, which is given as:

$$bias = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{\sigma}_z - \sigma_z\right) \qquad \text{(Eq. 4)}$$

Table 1. Properties and initial conditions of the soils analysed (mean values for 0.3-0.7 m depth). F_{wheel}, wheel load; P_{tyre}, tyre inflation pressure; Clay < 0.002 mm; Silt 0.002–0.05 mm; Sand 0.05–0.2 mm; ρ_0, initial bulk density; w_0, initial gravimetric water content; σ_{pc}, precompression stress; *v*, estimated concentration factor.

Site	F_{wheel}	P_{tyre}	Clay	Silt	Sand	ρ_0	w_0	σ_{pc}	*v*
	kN	kPa	----- % by weight -----			Mg m^{-3}	g g^{-1}	kPa	
Billeberga (S)	86	100, 150, 250	30.6	40.2	29.2	1.68	0.189	92.3	3.3
Önnestad (S)	82	90, 220	35.0	48.4	16.7	1.54	0.250	138.4	2.8
Strängnäs (S)	32	160, 220	61.0	30.7	8.3	1.39	0.332	129.9	5.8
Ultuna (S)	11, 15, 33	70, 100, 150	60.6	23.8	15.7	1.40	0.311	73.0	3.1
Vallø (DK)	24	60	13.3	26.8	60.0	1.56	0.175	96.8	4.5
Mean									3.9

The sensitivity of the simulated soil stress and hence *v* to the upper model boundary condition was investigated by running simulations using (i) measured stress distribution at 0.1 m depth, i.e. near the tyre-soil contact, (ii) estimated stress distribution as described above, and (iii) different commonly-used approximations of the tyre-soil contact stress distribution such as uniform or power-law distributions (3).

Simulations using a finite element model

Additional simulations were carried out using finite element modelling (FEM) within the framework of COMSOL Multiphysics Version 4.2. The aim was to investigate the influence of elasto-plastic material properties and soil layers (topsoil over plough pan over subsoil) of different stiffness and strength on propagation of vertical stresses.

We applied a surface pressure of 250 kPa acting on a circular area of 0.5 m radius. The model was formulated as an axisymmetric problem (5 m radius, 5 m depth). The mesh was vertically divided into three layers (plough layer: 0-0.25 m depth, plough pan: 0.25-0.35 m depth; subsoil: 0.35-5 m depth) for which different mechanical properties could be defined. A bi-linear elasto-plastic model with strain hardening was chosen as a constitutive relationship (for details, see 17). Soil mechanical properties were adopted from the studies on "Ruckfeld" silt loam soil (18, 19) (Table 2). Young's modulus was calculated according to (20), and the isotropic tangent modulus was estimated as one-tenth of Young's modulus.

Results

Accuracy of stress transducers

The comparison between F_{wheel} and F_{app} revealed that the transducers overestimated the true stress by 10% on average. Consequently, measured soil stress values were corrected (reduced) by 10% for further analysis. Further details will be presented in (8).

Table 2. Mechanical properties of "Ruckfeld" silt loam soil (18, 19). Note that indices (b), (c) and (d) correspond to the notation on the stress calculations for the different "layering scenarios" shown in Fig. 2.

	Topsoil (0-0.25 m)	*Plough pan* (0.25-0.35 m)	*Subsoil* (0.35-5 m)
Bulk density [kg m^{-3}]	1.3×10^{3}	1.5×10^{3}, $1.6\times10^{3(c,d)}$	1.5×10^{3}
Young's Modulus [kPa]	1500	$3\times10^{3(b)}$, $5\times10^{3(c)}$,$5\times10^{5(d)}$	3×10^{3}
Poisson's ratio [-]	0.33	0.33	0.33
Precompression stress [kPa]	40	$80^{(b)}$, $150^{(c)}$, $300^{(d)}$	80
Isotropic tangent modulus [kPa]	150	$300^{(b)}$, $500^{(c)}$, $5\times10^{4(d)}$	300

$^{(b)}$ Same mechanical properties for plough pan and subsoil; $^{(c)}$ Plough pan according to the measurements by Berli et al. (18, 19); $^{(d)}$ Plough pan with twice the precompression stress and 100 times the stiffness of (c)

Estimation of the concentration factor, v

The average v per soil was in the range 2.8 to 5.8, with a mean value of 3.9 (coefficient of variation, C.V. = 33%), see Table 1. The lowest value for v of 2.8 was obtained for loading with 82 kN wheel load on a silty clay loam ('Önnestad'). The highest value for v of 5.8 was obtained for loading with 32 kN wheel load on a clay soil ('Strängnäs'). The RMSE (Eq. 2) was in the range 6.4 to 23.0 kPa, with a mean value of 15.0 kPa. The average bias (Eq. 4) was negative (-9.7 kPa; i.e. underestimation of stress) at 0.3 m depth (i.e. the uppermost sensor depth), positive (12.3 kPa; i.e. overestimation of stress) at 0.5 m depth (i.e. the intermediate sensor depth), and close to zero (-1.1 kPa) at 0.7 m depth (i.e. the lowest sensor depth).

The average value for v of 3.9 was not significantly different ($p > 0.05$) from $v = 3$. For $v = 3$, Eq. (1) satisfies the elastic theory of Boussinesq (1).

Sensitivity of the concentration factor to measured stress

The sensitivity of v to the values of measured stress was tested by comparing estimates of v using corrected stress readings (see above) with estimates of v using raw data (not corrected stress readings). We also estimated v when using the corrected stress values reduced by 10%.

When using uncorrected values (raw data) of measured stress, the average v was 4.7, i.e. 21% higher than when using the correct estimates of stress ($v = 3.9$). On the other hand, if the stress was 10% lower than the true stress (corrected stress), then we obtained an average value for v of 3.2, which is 17% lower than when using the correct estimates of stress. Hence, an uncertainty in measured stress of ±10% resulted in an uncertainty in v of roughly ±20%.

Sensitivity of the concentration factor to the upper model boundary condition

The sensitivity of v to the upper model boundary condition was evaluated using the soil and one loading situation (86 kN wheel load, 150 kPa tyre inflation pressure) at 'Billeberga' (Table 1, Fig. 1), because the measured stress distribution at 0.1 m depth was available for that (12).

We estimated v = 3.6 when simulating with a stress distribution that was estimated from tyre and loading characteristics by means of the model by Keller (16). If we instead used the measured stress distribution at 0.1 m depth (12), we obtained a value for v of 3.8, which is only slightly different from the v estimate of the former simulation. The RMSE and the bias of the two simulations were very similar. Furthermore, we run simulations assuming an elliptical contact area and either a uniform or a power-law distribution (using either a power of 1.5 or 2). All these shapes of (theoretical) stress distributions are frequently used as approximations of the real stress distribution at the tyre-soil contact (3). The estimates of v were 5.0 (uniform stress distribution), 3.2 (power-law distribution with a power of 1.5), and 2.5 (power-law distribution with a power of 2, i.e. parabolic distribution). For the uniform distribution, the RMSE was 29.7 kPa, which is about twice as high as for the simulations using either measured (RMSE = 14.0 kPa) or model-estimated stress distribution (RMSE = 13.3 kPa), and the bias was highly negative at 0.3 m depth (-48.7 kPa). The parabolic stress distribution resulted in a positive bias at 0.3 m depth (9.5 kPa) and an RMSE of 17.9 kPa. For the case of the power-law stress distribution with a power of 1.5, the RMSE and the bias were similar as for the simulation with the measured or model-estimated stress distributions. The different simulations are shown in Fig. 1.

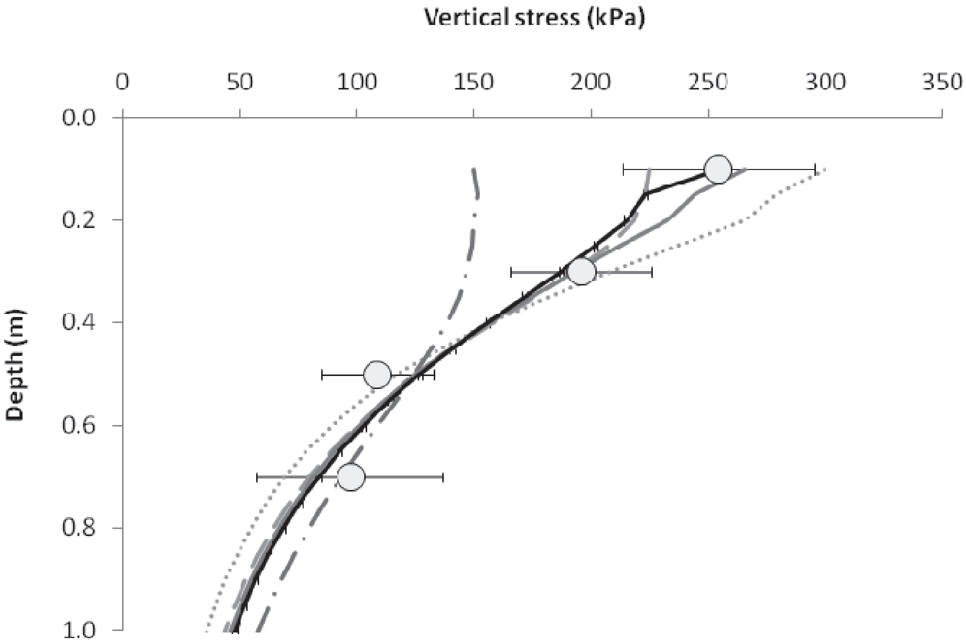

Figure 1. Measured (circles) and simulated vertical stress beneath the centre of a wheel with a 1050/50R32 tyre with a load of 86 kN and an inflation pressure of 150 kPa using uniform stress (chain dotted curve; v = 5.0, RMSE = 29.7), parabolic distribution (dotted curve; v = 2.5, RMSE = 17.9), power-law distribution with a power of 1.5 (dashed curve; v = 3.2, RMSE = 14.3), calculated stress distribution with the model of Keller (16) (grey curve; v = 3.6, RMSE = 13.3) and measured stress distribution (black curve; v = 3.8, RMSE = 14.0) as model input at 0.1 m depth. Error bars indicate standard deviation. See text for details.

Effects of soil layering on stress propagation by means of finite element model simulations

Figure 2a compares the numerical FEM calculations with the exact analytic solution based on Boussinesq's equation (1) providing a calibration of the FEM model for a linear-elastic, homogeneous, isotropic soil. Figure 2b shows a comparison of the analytic solution from Fig. 2a with an FEM calculation for elasto-plastic soil consisting of a plough layer (0-0.25 m depth) over a subsoil (0.25-5 m depth) without a plough pan (for details on the material properties, see Table 2). Although for the given applied load plastic deformation occurred in the topsoil and the upper part of the subsoil, vertical stress profiles are very similar for elastic and elasto-plastic soil. Also the layering (soft plough layer over stiffer subsoil, Table 2) had no influence on the vertical stress profile. Figure 2c shows similar calculations as in Fig. 2b but considering a 0.1 m thick plough pan in 0.25-0.35 m depth between the plough layer and the subsoil. Values for plough pan stiffness (5000 kPa) and precompression stress (130 kPa at 6hPa suction) were derived from actual measurements for "Ruckfeld" soil (18, 19). The vertical stress profiles in Fig. 2c are very similar to the one in Figs. 2a and b, indicating that a plough pan of "normal" stiffness (and 0.1 m thickness) has little to no effect on the vertical stress profile. Figure 2d shows similar calculations as in Fig. 2c but for a plough pan 100 times stiffer (Young's Modulus of 500 MPa instead of 5 MPa) and with considerably higher precompression stress (300 kPa instead of 130 kPa) than for the calculations in Fig. 2c. For the case of a very stiff plough pan, vertical stress within and immediately below the plough pan is decreased compared to the stress profiles from Figs. 2a-c. Although theoretically possible, a Young's modulus of 0.5 GPa seems to be unrealistically high for a "soft" porous material such as agricultural soil at field capacity. The precompression stress value of 300 kPa was chosen so that the plough pan did not yield under the given load.

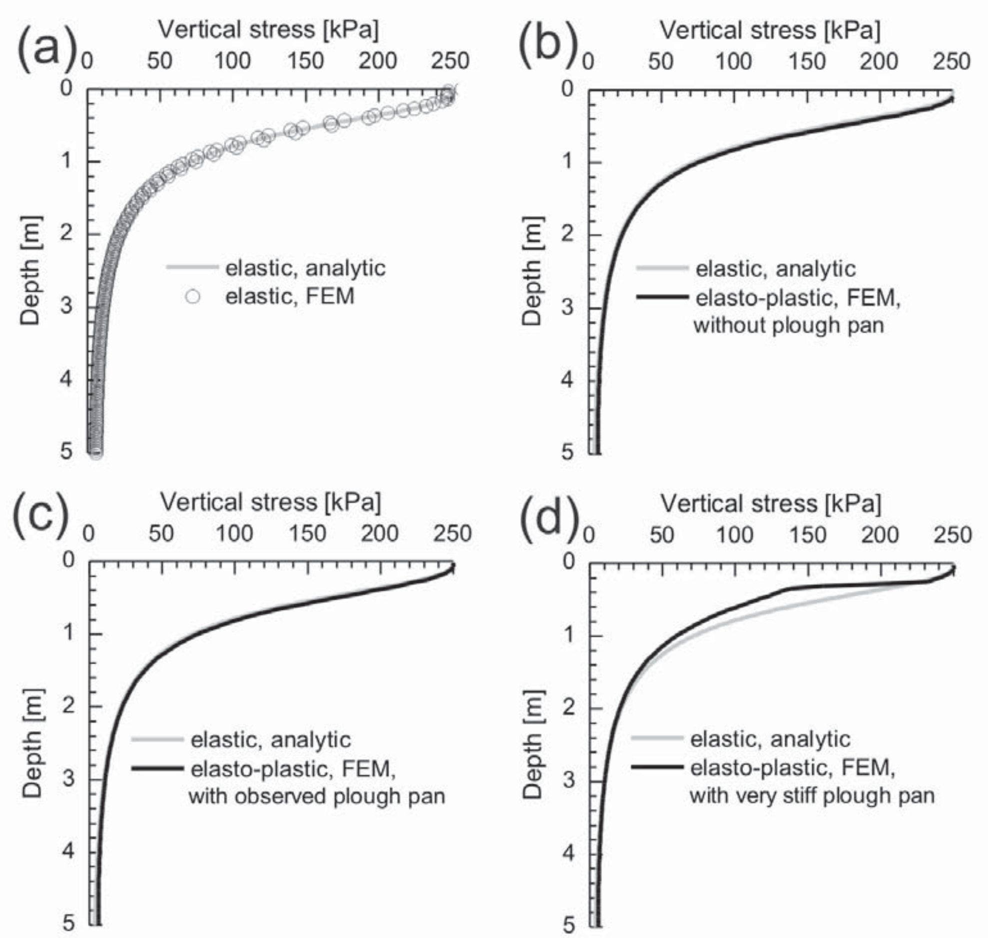

Figure 2. Calculated vertical stress as a function of depth. (a) Comparison of analytic with finite element model (FEM) calculations for linear-elastic, homogeneous, isotropic soil. (b) Comparison of the analytic solution from (a) with FEM calculation for an elasto-plastic soil consisting of a topsoil (0-0.25 m depth) over a subsoil (0.25-5 m depth) without a plough pan. (c) Comparison of the analytic solution from (a) with FEM calculation for an elasto-plastic soil consisting of a soft topsoil (0-0.25 m depth), a ploughpan with usually observed stiffness (0.25-0.35 m depth) over a subsoil (0.35-5 m depth). (d) Comparison of the analytic solution from (a) with FEM calculation for an elasto-plastic soil consisting of a topsoil (0-0.25 m depth), a very stiff plough pan [0.25-0.35 m depth, 100 times the stiffness of the plough pan in (c)] over a subsoil (0.35-5 m depth).

Discussion

Fröhlich's model including v is widely used in agricultural soil mechanics, usually in the form of Eq. (1) (7). Despite the wide use of this model, little is known about v, e.g. how it may vary with soil type and conditions.
Fröhlich (2) made the assumption that stress propagates along straight lines through the soil. The effect of v can be seen in analogy to the propagation of light in vacuum or air from an infinitively small light source. Similar to the focussing effect of a lens for light, Fröhlich assumed that the soil, depending on its properties, will have a focussing effect on the 'stress beams', which he expressed by v. Depending on the soil properties, stress beams were considered to be more or less focussed towards the centre line of the point load.
It is commonly accepted that v increases with decreasing soil strength (3, 21). For example, Söhne (3) suggested that v takes values of 4, 5 and 6 for hard, firm and soft soil, respectively. According to Horn (21), v is not only influenced by soil properties and conditions, but also affected by the applied load. Results from Lamandé et al. (22) indicate that soil deformation influences v. Several researchers have presented estimates of v for different soil and loading conditions. Dexter et al. (23) and Horn (21) presented values for v in the range 0.6-4.6 and 1.1-4.7, respectively, i.e. lower than those given by Söhne (3). Keller and Lamandé (7) obtained values for v between 4.6 and 7.7, which includes higher values than those proposed by Söhne (3). Even higher values for v in the range 6.4 to 14.3 were reported (24, 25, 26).
However, as shown in this paper, the estimation of v is strongly dependent on the accuracy of stress measurements, and on the upper stress boundary condition used for simulations.
The accuracy of stress transducers, i.e. the relation between measured stress and actual/true soil stress, and hence the accuracy of stress measurements is not well known (6, 7). The stress estimate provided by a transducer is influenced by a range of factors including transducer dimensions and the mechanical properties of the transducer in relation to those of its surrounding soil (5, 6, 27). The interaction of the different factors affecting stress readings is complicated, and therefore, there is no general means of correcting them (5). In the present study, it was concluded based on field measurements that the transducers used for stress measurements overestimate the true stress by 10%. The 10% are within the range of the modelling results for vertical transducers reported by Kirby (5).
The upper model boundary condition includes the magnitude and distribution (shape) of stress applied at the soil surface (e.g. the stress distribution at the tyre-soil contact area), and the area over which the load is applied (e.g. the tyre-soil contact area), and forms the input into Eq. (1). As shown by Keller (16), the upper model boundary condition is of paramount importance for accurate prediction of the stress propagation in soil. Unfortunately, the upper model boundary conditions is (i) typically not know *a priori*, and (ii) governed by a complicated interaction of tyre and soil properties (7). In this paper, we show that the estimate of v varies greatly (e.g. between 2 and 5 for the example presented, see Results section) just by applying different stress distributions (but always the same load) at the soil surface.
Based on this, the question is allowed whether a concentration factor is needed because the classical Boussinesq solution (i.e. Eq. (1) with $v = 3$) is insufficient to represent stress propagation in soil, or whether a concentration factor was introduced because the measurements were inaccurate (4). It is interesting to note that for the conditions investigated in this paper (Table 1), the average value for the estimated v of 3.9 was not significantly different ($p > 0.05$) from $v = 3$, i.e. the classical Boussinesq solution, when accounting for realistic upper model boundary condition and accurate stress measurements.
Selvadurai (28) observed that the solution provided by Fröhlich (2) satisfied (i) the equations of static equilibrium, globally and locally, (ii) the traction boundary conditions on the free surface, (iii) the regularity of decay of stress and displacement fields applicable to semi-infinite domains (i.e. decay of energy transfer), (iv) the equations of elasticity applicable to a homogeneous incompressible elastic material, *but* (v) violated the Beltrami-Michell equations of compatibility (29) applicable to classical elastic continua, *except* when $v = 3$, which corresponds to Boussinesq's classical solution. The consequences of violation of the compatibility conditions results in a non-unique evaluation of the displacement fields from the four linear partial differential equations applicable to a state of axial symmetry.
Obviously, agricultural soil is neither homogeneous nor completely elastic, and therefore, the assumptions on which the classical Boussinesq solution (i.e. Eq. (1)

formulated for a point load and with v = 3) is based are violated. However, elastic solutions may provide satisfactory approximations well beyond the range of small-deformation, linear-elastic material behaviour (20). Furthermore, results from simulations by FEM (Fig. 2) indicate that for a layered soil (topsoil over plough pan over subsoil) propagation of vertical stresses is not appreciably different from that in homogeneous soil unless unrealistically high differences in soil stiffness are considered However, we note that patterns of stress decay with depth have been reported in the literature than can probably not be reproduced by Eq. (1) (24, 25, 26, 30, 31). We suggest that further research on stress propagation in arable soil is needed.

Conclusions

This paper demonstrates that the comparison between measured and simulated soil stress is not an easy exercise, because (i) measurements of stress in soil may be biased because transducers do not read true stresses (but the accuracy of the stress transducers is normally not known), and (ii) the performance of simulations of soil stress is highly affected by the magnitude and distribution of the applied stress (e.g. the tyre-soil contact stress), i.e. the upper stress boundary condition (which is typically unknown *a priori*).

We investigated the accuracy of stress transducers used in this study, i.e. the relation between measured stress and true soil stress, and found that the transducer readings over-predicted the true vertical stress by 10%.

We measured stress in the soil profile at 0.3, 0.5 and 0.7 m depth during wheeling at field capacity on five soils (13-66% clay), and simulated stress propagation with the semi-analytical model (2). The "concentration factor" (v) was obtained at minimum deviation between measurements and simulations. For the five soils, we obtained an average v of 3.9 (for stress propagating from 0.1 to 0.7 m depth). It is interesting that this was not significantly different from v = 3, i.e. not different from the classical Boussinesq solution (1). We noted that v was strongly dependent on the accuracy of stress measurements, and on the upper stress boundary condition used for simulations.

Finite element simulations indicated that for an elasto-plastic layered soil propagation of vertical stresses is not appreciably different from that in a homogeneous isotropic linear-elastic soil unless layers with (unrealistically) high soil stiffness are considered.

Our results highlight the importance of accurate stress readings and realistic upper model boundary conditions, and suggest that stress propagation could be described by the elastic theory for the conditions investigated.

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References

(1) Boussinesq, J., 1885. *Application des Potentiels à l'étude de l'équilibre et du Mouvement des Solides Élastiques*. Gauthier-Villars, Paris, 30 pp.

(2) Fröhlich, O.K., 1934. *Druckverteilung im Baugrunde*. Springer Verlag, Wien, 178 pp.

(3) Söhne, W., 1953. Druckverteilung im Boden und Bodenverformung unter Schlepperreifen. *Grundlagen der Landtechnik* 5, 49-63.

(4) Davis, R.O., Selvadurai, A.P.S., 1996. *Elasticity and Geomechanics*. Cambridge University Press, Cambridge.

(5) Kirby, J.M., 1999. Soil stress measurement: Part 1, transducer in a uniform stress field. *Journal of Agricultural Engineering Research* 72, 151-160.

(6) Berli, M., Eggers, C.G., Accorsi, M.L., Or, D., 2006. Theoretical analysis of fluid inclusion for in situ soil stress and deformation measurements. *Soil Science Society of America Journal* 70, 1441-1452.

(7) Keller, T., Lamandé, M., 2010. Challenges in the development of analytical soil compaction models. *Soil and Tillage Research* 111, 54-64.

(8) Lamandé, M., Keller, T., Berli, M., Berisso, F.E., Stettler, M., Bolduc, L., Ruiz, S., Schjønning, P. 2012. Accuracy of soil stress measurements: field experiment and finite element modelling (in preparation).

(9) Arvidsson, J., Andersson, S., 1997. Determination of soil displacement by measuring the pressure of a column of liquid. In: *Proc. 14th Int. Conf. ISTRO, Puławy, PL*, pp. 47-50.
(10) Schjønning, P., Lamandé, M., Tøgersen, F.A., Arvidsson, J., Keller, T., 2008. Modelling effects of tyre inflation pressure on the stress distribution near soil-tyre interface. *Biosystems Engineering* 99, 119-133.
(11) Keller, T., Arvidsson, J., 2007. Compressive properties of some Swedish and Danish structured agricultural soils measured in uniaxial compression tests. *European Journal of Soil Science* 58, 1373-1381.
(12) Keller, T., Arvidsson, J., 2004. Technical solutions to reduce the risk of subsoil compaction. *Soil and Tillage Research* 79, 171-205.
(13) Arvidsson, J., Traunter, A., Keller, T., 2002. Influence of tyre inflation pressure on stress and displacement in the subsoil. *Advances in Geoecology* 35, 331-338.
(14) Arvidsson, J., Keller, T., 2007. Soil stress as affected by wheel load and tyre inflation pressure. *Soil and Tillage Research* 96, 284-291.
(15) Keller, T., Défossez, P., Weisskopf, P., Arvidsson, J., Richard, G., 2007. *SoilFlex*: A model for prediction of soil stresses and soil compaction due to agricultural field traffic. *Soil and Tillage Research* 93, 391-411.
(16) Keller, T., 2005. A model for prediction of the contact area and the distribution of vertical stress below agricultural tyres from readily-available tyre parameters. *Biosystems Engineering* 92, 85-96.
(17) Shames, I.H., Cozzarellli, F.A., 1997. *Elastic and Inelastic Stress Analysis*. Taylor & Francis, Washington DC.
(18) Berli, M., Kirby, J.M., Springman, S.M., Schulin R., 2003. Modelling compaction of agricultural subsoils by tracked construction machinery under various moisture conditions. *Soil and Tillage Research* 73, 57-66.
(19) Berli, M., Kulli B., Attinger, W., Springman, S.M., Flühler, H., Schulin R., 2004. Compaction of agricultural and forest subsoils by tracked heavy construction machinery. *Soil and Tillage Research* 75, 37-52.
(20) Berli, M., Accorsi, M.L., Or, D., 2006. Size and shape evolution of pores in viscoplastic matrix under compression. *International Journal for Numerical and Analytical Methods in Geomechanics* 30, 1259-1281.
(21) Horn, R., 1990. Structure effects on strength and stress distribution in arable soils. In: *Proc. Int. Summer Meeting of ASAE, 24-27 June 1990, Columbus, Ohio*, pp. 8-20.
(22) Lamandé, M., Schjønning, P., Tøgersen, F.A., 2007. Mechanical behaviour of an undisturbed soil subjected to loadings: Effects of load and contact area. *Soil and Tillage Research* 97, 91-106.
(23) Dexter, A.R., Horn, R., Holloway, R., Jakobsen, B.F., 1988. Pressure transmission beneath wheels in soils on the Eyre peninsula of South Australia. *Journal of Terramechanics* 25, 135–147.
(24) Lamandé, M., Schjønning, P., 2011a. Transmission of vertical stress in a real soil profile. Part I: Site description, evaluation of Söhne model, and the effect of topsoil tillage. *Soil Tillage Research* 114, 57-70.
(25) Lamandé, M., Schjønning, P., 2011b. Transmission of vertical stress in a real soil profile. Part II: Effect of tyre size, inflation pressure and wheel load. *Soil and Tillage Research* 114, 71-77.
(26) Lamandé, M., Schjønning, P., 2011c. Transmission of vertical stress in a real soil profile. Part III: Effect of soil water content. *Soil and Tillage Research* 114, 78-85.
(27) Weiler, W.A., Kulhawy, F.H., 1982. Factors affecting stress cell measurements in soil. *Journal of the Geotechnical Engineering Division, Proceedings of the American Society of Civil Engineers* 108, 1529-1548.
(28) Selvadurai, A.P.S., 2012. On Fröhlich's result for Boussinesq's problem, *Proc. 83rd Annual Meeting of the Gesellschaft für Angewandte Mathematik und Mechanik*, Darmstadt, 405-406.
(29) Selvadurai, A.P.S., 2000. *Partial Differential Equations in Mechanics, Vol.2. The Biharmonic Equation, Poisson's Equation*, Springer-Verlag, Berlin.
(30) Richards, B.G., Peth, S., 2009. Modelling soil physical behaviour with particular reference to soil science. *Soil and Tillage Research* 102, 216-224.
(31) Trautner, A., Arvidsson, J., 2003. Subsoil compaction caused by machinery traffic on a Swedish Eutric Cambisol at different soil water contents. *Soil and Tillage Research* 73, 107-118.