Joint Routing, Link Scheduling and Power Control for Wireless Multi-hop Networks for CDMA/TDMA Systems

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1. Abstract

In this paper, we analyse the problem of joint routing, link scheduling and power control for wireless multi-hop networks for CDMA/TDMA systems. Our objective is to find the optimal route for a specific scheduling while reducing the interference between devices and increasing the single-hop throughput. We present the mathematical model that represents the network, which is a non convex non linear optimisation problem and solve it numerically.

Key words: wireless, multi-hop, CDMA/TDMA, routing, scheduling, power control.

2. Resumen (Optimización conjunta del ruteo, envío de información y control de potencia para sistemas de redes inalámbricas multi-salto para sistemas)

En este artículo, presentamos el análisis del problema de la optimización conjunta del ruteo, envío de información y el control de potencia para sistemas inalámbricas multi-salto para sistemas CDMA/TDMA. Nuestro principal objetivo es encontrar el ruteo óptimo en tiempos específicos mientras se reduce la interferencia entre los dispositivos y utilizando a su máxima capacidad el canal de comunicación. Para esto presentamos un modelo matemático que respresenta este tipo de redes. El modelo matemático aquí presente es un modelo de optimización no lineal y no convexo, el cual lo resolvemos numéricamente.

Palabras clave: wireless, multi-hop, CDMA/TDMA, routeo, scheduling, control de potencia.

3. Introduction

Nowadays most of the communication between two wireless devices is based on the use of a network infrastructure, i.e. there are base stations that controls and manage the communication to these wireless devices. This implies that the base station is responsible of the routing and scheduling for the messages transmission. Also in order to reduce the interference level over the network it is necessary to apply power control algorithms. These networks relay on the known OSI layered protocol. On the contrary, in an *ad hoc* multi-hop wireless networks the administration is carried out by those devices conforming the network. It is necessary for each device to detect those devices that are within its own transmission range, which will be called active connections, in order to route the message. Each active connection experiments interference from other active connections and background noise i.e. Signal to Interference Noise Ratio SINR. In the *ad hoc* wireless network the OSI protocol is not adequate since it is necessary the joint optimisation between different layers, which gives way to a cross layer protocol. This kind of network is capable of supporting high data rates. The minimization of the power is essential since all devices are powered by batteries with a finite lifetime.

A. Related Work

So far, many separate efforts have been made in order to solve these problems. For instance, routing problems based on single or multi-commodity flow assignment have been studied by Gallager, Stern and more recently by Grover [9, 11, 14], the power control has been studied by Yates, Foschini

and Grandhi [7, 10, 16, 17] and the scheduling problem has been studied by Makarevitch [12]. Although they have addressed the uses of routing, power control and scheduling, none of them has solved all three problems in a single solution. Ephremides and ElBatt [6] have considered the joint scheduling and power control in wireless systems problem by alternating two phases. Ephremides and Li present a similar idea in [18]. An interesting approach is presented by Johanson, Xiao and Boyd [15]. They formulate a simultaneous routing and resource allocation, assuming link capacity and resource allocations for TDMA/FDMA[3, 5] systems. Cruz and Santana [4] study the problem of joint routing, scheduling and power control for wireless multi-hop networks. The algorithm that they present computes an optimal link scheduling and power control policy that minimises the total average power transmission in the network. The algorithm presented is efficient, but it is only designed for single path routing. Kodialam and Bhatia in [1] present an interesting approach to the solution of the joint optimisation problem of routing, scheduling and power control for a TDMA/CDMA network.

In this paper we consider the jointly routing, scheduling and power control problem for CDMA/TDMA wireless networks. We present a solution for this non-convex problem. The two main contributions that we present in this paper are: the embedding of the time (scheduling) on the network topology and the numerical optimisation of the minimisation of the total power consumption in the network under the following considerations: routing of the message, channel capacity and Quality of Service (QoS). We do not consider the mobility of any devices neither devices entering or leaving the system. Since this problem is non-convex we cannot guarantee the global optimal solution, nevertheless we have proved that for any local minimiser the set of coupling constraints (5) that makes this problem non-convex are active. This is a very important contribution since this constraint represents the capacity of the channel, meaning that at this local minimal the capacity of the channel is used at its maximum capacity.

This paper is organized as follows: in Section 2 we present the mathematical model which describes *ad hoc* wireless networks and the used analysis framework. In Section 3 we present results for a numerical experiment and in the last section conclusions.

4. Development
4.1 Mathematical Model

Let us examine an *ad hoc* wireless network consisting of devices, which can communicate with each other via a wireless link. Without loss of generality, consider a message originated at a particular source and destined to arrive at a particular sink. The rest of the devices are considered as intermediate devices. The intermediate devices will be source or sink according to their function (sending or receiving mode) at a given time. The message is sent from the origin source through the network until it reaches its destination via an unknown path. The path is set by the connectivity and link transmission power and capacity. In this scheme there are three possibilities:

· The whole message is sent as a single piece.

· The message remains in the buffer of the device either because the device is in receiving mode or the capacity of the out going links are full, until it can be sent.

· The message is split into smaller pieces, depending on the links capacity. Then, each piece could be considered as a smaller message, which can be sent as a single piece, remain in the buffer or be split into smaller pieces.

Independently of how it is sent, the complete message will arrive at its destination integrated as a whole.

In this section we present the set of variables, the set of constraints and the objective function for the mathematical model of an *ad hoc* wireless network for CDMA/TDMA systems.

Let us consider an *ad hoc* wireless network consisting of r devices. Therefore, the set of nodes will be $V = \{v_1, ... v_r\}$ and the set of links E as defined by

$$E = \{e=(u,v) \in V \times V | \ u \neq v, \ \zeta_{u,v} \leq \rho_u\}$$

where $\rho_u \in \mathsf{R}_+$ is the transmission range of a device u and v the distance between any two devices u and v is $\zeta_{u,v} \in \mathsf{R}_+$.

In other words, there exists a link $e = (u,v) \in E$ from sender node $u \in V$ to receiver node $v \in V$ if and only if $u \neq v$ and node v is within the transmission range of node u.

The *ad hoc* wireless network, then can be represented by a digraph which is an ordered pair $G = (V, E)$ defined by the set of nodes V and the set of edges E. For the set of edges E, we define two subsets E^+ and E^- as follows:

For any node, $v \in V$, let us denote by

$$E^+(v) := \{e \in E | \exists \ u \in V : e = (v,u)\}$$

the set of edges that leaves the node v, and by

$$E^-(v) := \{e \in E \mid \exists\, u \in V : e = (u,v)\}$$

the set of edges that enters the node v, i.e. $E^+(v)$ is the set of outgoing edges from node v, and $E^-(v)$ is the set of incoming edges to node v.

Let us define the set of messages $M=\{m_1, ...,m_L\}$, where L is the number of messages to be sent along the network and let $z_{m_i} \in R_+$, $i=1,...,L$ be the size of the message m_i. For each message $m_i \in M$, $i=1,...,L$ there exist a source node $s_{m_i} \in V$ and a sink node $d_{m_i} \in V$ with $s_{m_i} \neq d_{m_i}$ exist. Since there are different senders and receivers in the network, we face a multi-commodity flow problem.

The time interval under consideration is equally divided into transmission slots. Define the set of consecutive time slots as $T = \{t_1,...,t_{tMax}\}$, where $t_t \in R^+$, $i \in 1,..., t^{Max}$ is a time interval whose end point is the starting point t_{i+1}.

We include a colouring scheme to solve the scheduling problem under the following definitions:

Given a set of colours $C=\{\alpha_1,...\alpha_k\}$, where k is the number of available colours, assign to each node $v \in V$ a colour according to the following mapping $Co_v: V \to C$ such that $Co_v(v) \neq Co_v(u)$ with $(u,v) \in E$ where

$$V_{\alpha_i} = \{v \in V : Co_v(v) = \alpha_i\}$$

for $\alpha_i \in C$. In particular k is chosen in such a way that

$$V = \bigcup_{i=1}^{k} V_{\alpha_i}$$

Furthermore, define the edge-colouring mapping $Co_e: E \to C$ by

$$Co_e: (u,v) \in E \to Co_e: (u,v) := Co_v(u)$$

where Co_v is a corresponding node-colouring and the time interval colouring mapping $Co_t: T \to C$ by

$$Co_t: t_i \in T \to Co_t: (t_i) = \alpha_j \text{ and } j = i\%k$$

(i.e. $Co_t(t_i) = \alpha_i$, $Co_t(t_{k+1}) = \alpha_t$, $i = 1,...,k$, etc.) where % is the modulo operation.

A. Design variables

A message m to be sent through an *ad hoc* wireless network may travel as a complete message or it may be split. The size of the part of that message m going through the edge e in time slot t is denoted by the decision variable $c_{e,m,t} \in R_*$.

The size of that part of the message m that is stored in the buffer of node v, at the beginning of the time slot t is denoted by $b_{v,m,t} \in R_*$. Moreover, at the beginning of the first time slot t_1 we have all messages m_i are stored in the buffer of the source nodes s_{m_i}. Nodes that are not source nodes have an empty buffer at the beginning of t_1. This is described by the following set of equations.

$$b_{v,m_i,t_1} = \begin{bmatrix} z_{m_i} & \text{if } v = s_{m_i} \\ \\ 0 & \text{else} \end{bmatrix} \tag{1}$$

The transmit power allocated at the edge e at the time slot t is denoted by $p_{e,t} \in R_*$.

B. Parameters

The size of the buffer at node v is denoted by $B_v^{Max} \in R_+$, the maximum transmission power available for all edges by $P^{Max} \in R_+$ and the maximum transmission power for each node v by $P_v^{Max} \in R_+$.

The starting time slot $t_{j(m)} \in T$ is the specific time slot at which the message is released (made available) to the user and $R_{m_i} > 0$ is the average rate at which the message is released, from the starting time slot $t_{j(m)} \in T$ onwards until the complete message is received by the user.

The maximum number of time slots t^{Max} needed for the complete reception of all messages is given by:

$$t^{Max} = \max \{t_{j(m_i)} + (z_{m_i}/R_{m_i}) \mid m_i \in M \}$$

The bandwidth of the communication channel is $B \in R_+$ and $\eta \in R_+$ is the background noise (measured in watts or dBm).

C. Set of constraints

Upper Bounds for the Decision Variables: The transmission power $p_{e,t}$ cannot exceed the maximum power P^{Max},

$$p_{e,t} \leq P^{Max} \qquad \forall t \in T \tag{2}$$

For any node v and for any time slot t, the sum of all transmission powers $p_{e,t}$ where e is varying in $E^+(v)$ cannot exceed the maximum power P_v^{Max}:

$$\sum_{e \in E^+(v)} p_{e,t} \leq P_v^{Max} \qquad \forall t \in T \tag{3}$$

The sum of the sizes of those parts of the messages stored at the beginning of the time slot t in the buffer of cannot exceed

the size of this buffer:

$$\sum_{m \in M} b_{v,m,t} \leq B_v^{Max} \quad \forall t \in T \tag{4}$$

The capacity of a channel (edge) is given by Shannon's well-known formula

$$\sum_{m \in M} c_{e,m,t} \leq B \log_2 \left[1 + \frac{\rho_e p_{e,t}}{\sum_{f \neq e} \rho_f p_{f,t} + \eta^2} \right] \quad \forall t \in T \tag{5}$$

where ρ_e denotes the gain for the edge e. This set of constraints implies that the sum of all sizes of the parts of the messages $c_{e,m,t}$ cannot exceed the maximum capacity of the channel.

Constraints of the System: The following constraint describes the scheduling process by assigning the value zero to the decision variable $c_{e,m,t}$ when the edge colouring $Co_e(e)$ for the edge e is different from time colouring $Co_t(t)$:

$$c_{e,m,t} = 0 \qquad (m \in M, \; Co_e(e) \neq Co_t(t)) \tag{6}$$

In other words, $c_{e,m,t} \neq 0$ implies that the edge e has the same colour as the time slot t.

The relationship between the size $b_{v,m,tl}$ and $b_{v,m,tl+1}$ ($l = 1,...,t^{Max}-1$) of those parts of message m stored in the buffer of v at the beginning of time slots t_l and t_{l+1} respectively and the corresponding ingoing and outgoing message in v is represented by:

$$b_{v,m,t_{l+1}} - b_{v,m,t_l} =$$

$$b_{v,m,t_1} + \sum_{e \in E^-(v)} \sum_{i=1}^{l} c_{e,m,t_i} - \sum_{e \in E^+(v)} \sum_{i=1}^{l} c_{e,m,t_i}$$

$$- \left[b_{v,m,t_1} + \sum_{e \in E^-(v)} \sum_{i=1}^{l-1} c_{e,m,t_i} - \sum_{e \in E^+(v)} \sum_{i=1}^{l-1} c_{e,m,t_i} \right]$$

$$= \sum_{e \in E^-(v)} c_{e,m,t_l} - \sum_{e \in E^+(v)} c_{e,m,t_l} \tag{7}$$

At the sink node d_m, the message m will be stored in its buffer and will be released from it to the user at a constant reduction rate R_m, from the starting time slot $t_{j(m)} \in T$ onwards until the complete message is released. This is described by the following set of constraints

$$\sum_{i=1}^{t_{j(m)}+\iota} \sum_{e \in E^-(d_m)} c_{e,m,t_i} \geq (\iota + 1) R_m$$
$$(\iota = 0,..., [Z_m/R_m] - 1) \tag{8}$$

Non-Negativity Constraints: The following constraints refer to the non-negativity of the variables.

$$p_{e,t} \geq 0 \tag{9}$$

$$c_{e,m,t} \geq 0 \tag{10}$$

$$b_{v,m,t} \geq 0 \tag{11}$$

D. The non-linear programming problem

The objective function is to minimise the total power consumption in the system, with respect to all variables $p_{e,t}, c_{e,m,t}, b_{v,m,t}$ and is defined as follows:

$$f(p_{e,t}, c_{e,m,t}, b_{v,m,t}) = \sum_{e \in E} \sum_{t \in T} p_{e,t} \tag{12}$$

Note that this function is monotonically increasing with respect to $(p_{e,t})_{e \in E, t \in T}$.

As a matter of simplicity, we define the vectors $\mathbf{p} \in \mathsf{R}^{|E|*|T|}$, $\mathbf{c} \in \mathsf{R}^{|E|*|M|*|T|}$, $\mathbf{b} \in \mathsf{R}^{|V|*|M|*|T|}$ whose components are $p_{e,t}, c_{e,m,t}, b_{v,m,t}$:

$$\mathbf{p} = (p_{e,t})_{e \in E, t \in T}$$
$$\mathbf{c} = (c_{e,m,t})_{e \in E, m \in M, t \in T}$$
$$\mathbf{b} = (b_{v,m,t})_{v \in V, m \in M, t \in T}$$

Now, we can define the non-linear programming problem Θ:

minimise $f(\mathbf{p},\mathbf{c},\mathbf{b}) = \sum_{e \in E} \sum_{t \in T} p_{e,t}$ subject to (1)-(11).

Denote the feasible set of Θ by S. Obviously, S is a subset of $\mathsf{R}^{|E|*|T|+|E|*|M|+|T|+|V|*|M|+|T|}$.

It is obvious that the set of constraints 5 is non-convex, therefore this optimisation problem is non-convex, so at the best there exist local minimisers which must be found numerically. We have proved that if there exist a local minimiser then the set of constraints 5 are active at this point the Appendix [13].

4.2 Numerical experiment and solution

In order to solve the following we use AMPL[8] to model the non-linear problem Θ and the solver Knitro [2]. A contribution to this work is the embedding of the scheduling problem into the network topology. This procedure helped us to visualise the behaviour of the message flowing through the network at a certain time slot. For more information about the embedding see [13].
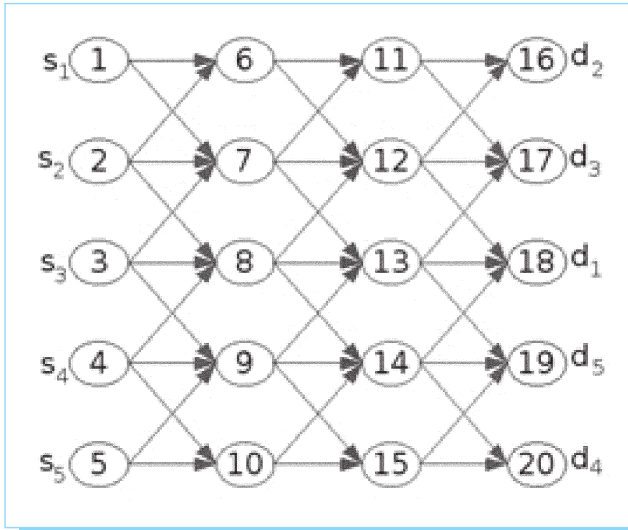
Fig. 1. Five messages network topology.

Consider five messages m_1, m_2, m_3, m_4, m_5 to be sent along the digraph $G(V,E)$. The graphical representation $G(V,E)$ is given in Figure 1.

Figure 1 shows the source nodes s_m and the sink nodes d_m for each message m. For example $s_{m_1} = v_1$ is the source node of message and the sink node for message m_1 is $d_{m_1} = v_{18}$. The size of each message is 500 Kbits and the reduction rate at each sink node is 100 Kbits, we have $z_{m_i} = 500$, $R_{m_i} = 100$, $i \in 1,...,5$. We solved the non-linear programing problem Θ for five messages being sent through $G(V,E)$ with the set of parameters given in table I.

After 33 iterations executed in 0.4 seconds, the solver Knitro found a local minimiser with objective function value of 0.01022744 mW.

Table 1. List of parameters.

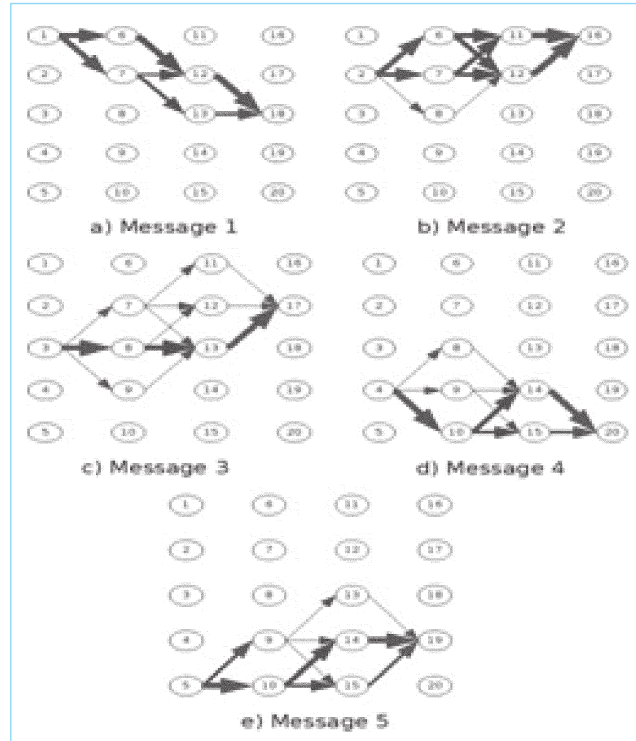| Parameter	Value	Units
P_v^{Max}	2	W
B_v^{Max}	4 000	Kbit
$P_{e,m}^{Max}$	2	W
η	0.008	mW
B	5	MHz

Fig. 2. The flows for each of the five messages.

In Figure 2, the thickness of each arrow represents the size of information flowing through the edge. As we mentioned before the maximum capacity of the channel is guaranteed since the set of constrains 5 are active for any local minimiser the mathematical prove of this is presented in the Appendix. The results for p_e and $c_{e,m}$ at this local minimiser are shown in the following tables.

5. Conclusions

We solved numerically the non-convex problem Θ which presents an integrated solution of the joint routing, scheduling and power control problem for a multi-hop wireless network. A contribution presented here is the embedding of the scheduling problem into the network topology. This procedure helped us to visualise the behaviour of the message flowing through the network at a certain time slot. We have seen that our model can support higher data transmission with minimum power consumption.

Appendix

Let $(\bar{p}, \bar{c}, \bar{b})$ be a local minimiser of the problem Θ. Then, the constraints (5) are active at $(\bar{p}, \bar{c}, \bar{b})$ i.e.

Table 2. Transmission power assigned to each edge.

Edge (v_i, v_j)	Power $P_{(vi, vj)}$	Edge (v_i, v_j)	Power $P_{(vi, vj)}$
(v_1, v_6)	0.00090369	(v_8, v_{13})	0.0005612
(v_1, v_7)	0.00262205	(v_8, v_{14})	0.0008661
(v_2, v_6)	0.00137324	(v_9, v_{13})	0.0015993
(v_2, v_7)	0.00144031	(v_9, v_{14})	0.0013717
(v_2, v_8)	0.00077044	(v_9, v_{15})	0.0014213
(v_3, v_7)	0.00188902	(v_{10}, v_{14})	0.0012529
(v_3, v_8)	0.00103666	(v_{10}, v_{15})	0.0012972
(v_3, v_9)	0.00064796	(v_{11}, v_{16})	0.00142285
(v_4, v_8)	0.00086544	(v_{11}, v_{17})	0.00072822
(v_4, v_9)	0.00137037	(v_{12}, v_{16})	0.00213245
(v_4, v_{10})	0.00135036	(v_{12}, v_{17})	0.00115653
(v_5, v_9)	0.00234516	(v_{12}, v_{18})	0.00220283
(v_5, v_{10})	0.00119760	(v_{13}, v_{17})	0.00170067
(v_6, v_{11})	0.0007858	(v_{13}, v_{18})	0.00135113
(v_6, v_{12})	0.0014920	(v_{13}, v_{19})	0.00096986
(v_7, v_{11})	0.0013658	(v_{14}, v_{19})	0.00131290
(v_7, v_{12})	0.0027324	(v_{14}, v_{20})	0.00214463
(v_7, v_{13})	0.0018488	(v_{15}, v_{19})	0.00130886
(v_8, v_{12})	0.0012440	(v_{15}, v_{20})	0.00141074

Table 3. Sizes of pieces for message m_1.

Edge (v_i, v_j)	$c_{(vi, vj)m_1}$
(v_1, v_6)	406.39
(v_1, v_7)	1093.61
(v_6, v_{12})	406.39
(v_7, v_{12})	529.9
(v_7, v_{13})	563.72
(v_{12}, v_{18})	936.29
(v_{13}, v_{18})	563.71

Table 4. Sizes of pieces for message m_2.

Edge (v_i, v_j)	$c_{(vi, vj)m_2}$
(v_1, v_6)	574.05
(v_2, v_7)	595.05
(v_2, v_8)	330.91
(v_6, v_{11})	327.83
(v_6, v_{12})	246.21
(v_7, v_{11})	273.81
(v_7, v_{12})	321.24
(v_8, v_{12})	330.91
(v_{11}, v_{16})	601.65
(v_{12}, v_{16})	898.36

Table 5. Sizes of pieces for message m_3.

Edge (v_i, v_j)	$c_{(vi, vj)m_3}$
(v_3, v_7)	777.55
(v_3, v_8)	441.74
(v_3, v_9)	280.71
(v_7, v_{11})	299.76
(v_7, v_{12})	270.54
(v_7, v_{13})	207.24
(v_8, v_{12})	207.05
(v_8, v_{13})	234.69
(v_9, v_{13})	280.71
(v_{11}, v_{17})	299.76
(v_{12}, v_{17})	477.6
(v_{13}, v_{17})	722.64

Theorem 1:

$$\sum_{m \in M} \bar{c}_{e,m,t} = B \log_2 \left[1 + \frac{\bar{P}_{e,t}}{\sum_{f \neq e} \bar{P}_{f,t} + \eta^2} \right]$$

Proof. Suppose that there exist a local minimiser $(\widetilde{\mathbf{p}}, \widetilde{\mathbf{c}}, \widetilde{\mathbf{b}})$ of Θ and indices $\bar{e} \in E$, $\bar{t} \in T$ such that:

Table 6. Sizes of pieces for message m_4.

Edge (v_i, v_j)	$c_{(vi,vj)m_4}$
(v_4, v_8)	367.77
(v_4, v_9)	570.25
(v_4, v_{10})	561.98
(v_8, v_{14})	367.77
(v_9, v_{14})	272.46
(v_9, v_{15})	297.78
(v_{10}, v_{14})	268.75
(v_{10}, v_{15})	293.23
(v_{14}, v_{20})	908.98
(v_{15}, v_{20})	591.02

$$\sum_{m \in M} \widetilde{c}_{\bar{e},m,\bar{t}} < B \log_2 \left[1 + \frac{\widetilde{p}_{\bar{e},\bar{t}}}{\sum_{f \neq \bar{e}} \widetilde{p}_{f,\bar{t}} + \eta^2} \right] \qquad (13)$$

As $(\widetilde{\mathbf{p}}, \widetilde{\mathbf{c}}, \widetilde{\mathbf{b}})$ is a local minimiser of Θ there exists an $\bar{\varepsilon} > 0$ such that $f(\widetilde{\mathbf{p}}, \widetilde{\mathbf{c}}, \widetilde{\mathbf{b}}) \leq f(\mathbf{p}, \mathbf{c}, \mathbf{b}) \ \forall (\mathbf{p}, \mathbf{c}, \mathbf{b}) \in S \cap B_{\bar{\varepsilon}} (\widetilde{\mathbf{p}}, \widetilde{\mathbf{c}}, \widetilde{\mathbf{b}})$ \qquad (14)

From (13) we obtain:

$$\widetilde{p}_{e,t} > \left[2^{\frac{\sum_{m \in M} \widetilde{c}_{\bar{e},m,\bar{t}}}{B}} - 1 \right] \left[\sum_{f \neq \bar{e}} \widetilde{p}_{f,\bar{t}} + \eta^2 \right] \qquad (15)$$

Table 7. Sizes of pieces for message m_5.

Edge (v_i, v_j)	$c_{(vi,vj)m_5}$
(v_5, v_9)	980.26
(v_5, v_{10})	519.74
(v_9, v_{13})	397.34
(v_9, v_{14})	291.84
(v_9, v_{15})	291.08
(v_{10}, v_{14})	260.83
(v_{10}, v_{15})	258.91
(v_{13}, v_{19})	397.34
(v_{14}, v_{19})	552.67
(v_{15}, v_{19})	549.99

Let us define the right hand side of (15) as :

$$r := \left[2^{\frac{\sum_{m \in M} \widetilde{c}_{\bar{e},m,\bar{t}}}{B}} - 1 \right] \left[\sum_{f \neq \bar{e}} \widetilde{p}_{f,\bar{t}} + \eta^2 \right] \qquad (16)$$

By (9), (10) and (16), we have that $r \geq 0$.
Thus, (15) can be written as

$$\widetilde{p}_{e,t} - r > 0 \qquad (17)$$

which implies $\widetilde{p}_{\bar{e},\bar{t}} > 0$

Define $\varepsilon := \widetilde{p}_{\bar{e},\bar{t}} - r$ \qquad (18)

Choose a $\lambda \in (0,1)$ with $\lambda \bar{\varepsilon} < \varepsilon$ and define the feasible point $(\hat{\mathbf{p}}, \hat{\mathbf{c}}, \hat{\mathbf{b}})$ as:

$$\hat{\mathbf{c}} = \widetilde{\mathbf{c}}, \qquad \hat{\mathbf{b}} = \widetilde{\mathbf{b}}$$

$$\widehat{p}_{e,t} = \begin{cases} \widetilde{p}_{e,t}, & \text{if } (e,t) \neq (\bar{e}, \bar{t}) \\ \widetilde{p}_{e,t} - \lambda \varepsilon, & \text{if } (e,t) = (\bar{e}, \bar{t}) \end{cases} \qquad (19)$$

Obviously, $(\hat{\mathbf{p}}, \hat{\mathbf{c}}, \hat{\mathbf{b}})$ is a small perturbation of $(\widetilde{\mathbf{p}}, \widetilde{\mathbf{c}}, \widetilde{\mathbf{b}})$ and, by construction, we have

$$\widetilde{p}_{e,\bar{t}} > 0, \qquad \widehat{p}_{e,\bar{t}} > r \qquad (20)$$

Now, from (16) and (20) we have that $\sum_{f \neq \bar{e}} \widetilde{p}$

$$\widehat{p}_{\bar{e},\bar{t}} > \left[2^{\frac{\sum_{m \in M} c_{\bar{e},m,\bar{t}}}{B}} - 1 \right] \left[\sum_{f \neq \bar{e}} \widetilde{p}_{f,\bar{t}} + \eta^2 \right]$$

Since $0 \leq r < \widehat{p}_{\bar{e},\bar{t}} < p_{\bar{e},\bar{t}}$, the point $(\hat{\mathbf{p}}, \hat{\mathbf{c}}, \hat{\mathbf{b}})$ fulfils all the constraints (1)-(11) i.e. it is a feasible point of the problem Θ.

By construction $(\hat{\mathbf{p}}, \hat{\mathbf{c}}, \hat{\mathbf{b}})$ is a feasible point of the problem and the value of f at $(\hat{\mathbf{p}}, \hat{\mathbf{c}}, \hat{\mathbf{b}})$ is

$$\sum_{e \in E} \sum_{t \in T} \widehat{p}_{e,t} = \sum_{e \in E} \sum_{t \in T} \widetilde{p}_{e,t} - \lambda \varepsilon \quad \text{and, by } \lambda \varepsilon > 0 \text{ , we have that}$$

$$f(\hat{\mathbf{p}}, \hat{\mathbf{c}}, \hat{\mathbf{b}}) = \sum_{e \in E} \sum_{t \in T} \widehat{p}_{e,t} < \sum_{e \in E} \sum_{t \in T} \widetilde{p}_{e,t} = f(\widetilde{\mathbf{p}}, \widetilde{\mathbf{c}}, \widetilde{\mathbf{b}})$$

The latter inequality contradicts (14) and, hence, $(\widetilde{\mathbf{p}}, \widetilde{\mathbf{c}}, \widetilde{\mathbf{b}})$ is not a local minimiser of Θ, contradicting our initial assumption. Therefore, the constraints (5) have to be active i.e.

$$\sum_{m \in M} \overline{c}_{\bar{e},m,t} = B \log_2 \left[1 + \frac{\overline{p}_{e,t}}{\sum_{f \neq \bar{e}} \overline{p}_{f,\bar{t}} + \eta^2} \right] \qquad (21)$$

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171

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